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Review Article

A REVIEW ARTICLE ON MEASUREMENT OF VISCOSITY

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ABSTRACT

The coefficient of viscosity is concerned with the viscous forces generated by compression (or dilatation). In the absence of knowledge of its magnitude in liquids it has been customary in hydrodynamics to assume that the coefficient of dilatational viscosity, n′, could be approximated by the ideal gas value n =- 2n/3, where n is the coefficient of shear viscosity. A method has been developed for obtaining values for the dilatational viscosity which is based on Eckhart's theory of acoustical streaming; the non-periodic motion of the fluid in the vicinity of a sound source is dependent on the two coefficients of viscosity. The coefficient of dilatational viscosity was found to be positive in sign and greater in magnitude than the shear viscosity. For example, the dilatational viscosity of water was found to be 2.4 centipoise and that for carbon disulfide greater than 200 centipoise. There is no correlation between the magnitude of the shear and dilatational viscosities for the liquids studied. Temperature variation measurements on water show that the temperature dependence of dilatational and shear viscosity in this substance is identical.

Keywords: Centipoise, Temperature, Coefficient of viscosity, Magnitude.

INTRODUCTION

Viscosity is a measure of the resistance of a fluid to deform under shear stress. It is commonly perceived as flow behavior or resistance to pouring. Viscosity describes a fluids internal resistance to flow and may be thought of as a measure of fluid friction. Viscosity at final plays a key role in the processing stage! For certain liquids viscosity is a material constant that only depends on temperature and pressure. This group of materials is termed Newtonian liquids. All fluids, i.e., all liquids and gases, exhibit viscosity to some degree. Viscosity may be thought of as fluid friction, just as the friction between two solids resists the motion of one over the other but also makes possible the acceleration of one relative to the other (e.g., the friction between the wheels of an automobile and a highway), so viscosity resists the motion of a solid through a fluid but also makes it possible for a propeller or other device to accelerate the solid through the fluid.

Velocity gradient

When a fluid is moving through a pipe or a solid object is moving through a fluid, the layer of fluid in contact with the sides of the pipe or the surface of the object tends to be in the same state of motion as the object with which it is in contact; that is, the layer of fluid along the side of the pipe is at rest, while that in contact with the moving object is carried along at the same velocity as the object. If the difference in velocity between the fluid at the sides of the pipe and that at the center, or between the moving object and the fluid through which it is moving, is not too great, then the fluid flows in continuous, smooth layers; that is, the flow is laminar. The difference in velocity between adjacent layers of the fluid is known as a velocity gradient and is given by v/x, where v is the velocity difference and x is the distance between the layers. To keep one layer of fluid moving at a greater velocity than the adjacent layer, a force F is necessary, resulting in a shearing stress F/A, where A is the area of the surface in contact with the layer being moved.

Coefficient of viscosity

The ratio of the shearing stress to the velocity gradient is a measure of the viscosity of the fluid and is called the coefficient of viscosity η, or η=*Fx /Av.* The CGS unit for measuring the coefficient of viscosity is the poise. Experiments have shown that the coefficient of viscosity of liquids decreases with increasing temperature, while the coefficient of viscosity of gases increases with increasing temperature. In liquids an increase in temperature is associated with the weakening of bonds between molecules; since these bonds contribute to viscosity, the coefficient is decreased. On the other hand, intermolecular forces in gases are not as important a factor in viscosity as collisions between the molecules, and an increase in temperature increases the number of collisions, thus increasing the coefficient of viscosity. A striking result of the kinetic theory of gases is that the viscosity of a gas is independent of the density of a gas. Viscosity is the principal factor resisting motion in laminar flow. However, when the velocity has increased to the point at which the flow becomes turbulent, pressure differences resulting from eddy currents rather than viscosity provide the major resistance to motion.

Dynamic Viscosity

Dynamic viscosity is known by the Greek symbol η, (eta). It is sometimes also referred to as shear viscosity. Dynamic viscosity is obtained by reformulating Newton's Law.

$$\tau = \eta \cdot \dot{\gamma} \Rightarrow \eta = \frac{\tau}{\dot{\gamma}} \quad [\mathrm{Pa{\cdot}s}] = \left[\frac{\mathrm{Pa}}{\frac{1}{s}}\right]$$

The SI unit is pascal-second [Pa.s] or millipascal-second [mPa.s]:

- ✓ 1 Pa.s = 1000 mPa.s
- ✓ The SI unit is named after Blaise Pascal.

Other commonly used units are poise [P] or centipoise [cP]:

- ✓ 1 P = 100 cP
- ✓ This unit is named after Jean Poiseuille

Relation between units

1 cP = 1 m Pa.s

Kinematic Viscosity

Kinematic viscosity (v) describes a substance's flow behavior under the influence of Earth's gravity. It is dynamic viscosity divided by density (ρ) which is defined as mass per volume. The quantity mass carries the gravitational influence.

$$\nu = \frac{\eta}{\rho} \left[\frac{m^2}{s}\right] \qquad \rho = \frac{m}{V} \left[\frac{kg}{m^3}\right]$$

Kinematic viscosity is widely established due to historical reasons: Gravity as the driving force does not require any elaborate technical equipment. It is simply available everywhere on earth.

The SI unit is square-meters per second [m^2/s] or square-millimeters per second [mm^2/s]

1 m^2/s = 1 000 000 mm^2/s

The SI units can be derived from the equation for the kinematic viscosity

$$\left[\frac{m^2}{s}\right] = \left[\frac{Pa{\cdot}s}{\frac{kg}{m^3}}\right] = \left[\frac{\frac{N}{m^2}\cdot s}{\frac{kg}{m^3}}\right]$$

$$[N] = \left[\frac{kg{\cdot}m}{s^2}\right]$$

$$\left[\frac{m^2}{s}\right] = \left[\frac{kg{\cdot}m}{s^2 \cdot m^2} \cdot s \cdot \frac{m^3}{kg}\right]$$

Other commonly used units are stokes [St] or centistokes [cSt]

1 St = 100 cSt

- This unit is named after George G. Stokes.

Relation between units

1 cSt = 1 mm^2/s

Relative Viscosity

Relative viscosity is an important parameter when testing polymers in solutions. For most polymers there is a definite relationship between molar mass and viscosity. You can measure the viscosity to determine the molar mass. The higher the molar mass is, the more viscous the polymer solution is. Molar mass is one of the most important quality parameters of polymers. Though most polymer solutions are actually non-Newtonian fluids, they behave like Newtonian fluids as long as the applied shear rate is low enough. The relative viscosity is calculated by dividing the viscosity of the polymer solution (η) by the viscosity of the pure solvent (η_0). The unit is

$$\eta_r = \frac{\eta}{\eta_0} [1]$$

Measuring the viscosity

Measuring viscosity is an effective way to know the state (properties of matter) or fluidity of a liquid or gas. It plays an important role in the quality control and in various research and development stages of a wide range of industries, including Food, Chemical, Pharmaceutical, Petrochemical, Cosmetics, Paint, Ink, Coatings, Oil and Automotives. For example, the viscosity of a liquid is an important parameter for designing the piping in a plant or transporting crude oil or chemical agent through a pipeline. In the electronic engineering industry, photo resist fluid is used in the production processes of the printed circuit board, cathode-ray tube, and flat liquid crystal display. Controlling the viscosity of photo resist fluid is a crucial factor to determine the qualities, performance, and yields of finished products. Among those industries, it has been also recognized that controlling optimum viscosity reduces production costs.Viscosity, which is also called a viscosity coefficient, is a measure of a fluid's resistance to flow. It is the substance constant indicating the magnitude of the fluidity of a fluid. In general, viscosity is associated only with liquid. Gas also has viscidity, but it is a relatively in viscid fluid, the resistance to flow can be ignored. Almost all liquids are viscous fluids having viscidity. For example, when rotating a drum container filled with water on its vertical central axis, the water that was at rest in the beginning starts moving as it is dragged by the container inside wall and then whirls completely together with the container as if it were a single rigid body. This is caused by the force (resistance) generated in the direction of the flow (movement) on the surfaces of the water and the container's inside wall. A fluid that generates this kind of force is regarded as having viscosity. Viscous fluid is divided into two broad categories:

- ✓ Newtonian fluid, subject to Newton's law of viscosity, viscosity is constant regardless of the flow (movement).
- ✓ Non-Newtonian fluid, which is not subject to Newton's law of viscosity, viscosity changes according to the flow (movement).

Note: subject to a constant temperature.

Types of viscometers

Sine-wave Vibro Viscometer

The below figure-1 shows the basic mechanism of one of the vibro viscometers, A&D's Sine-wave Vibro Viscometer.When the spring plates are vibrated with a uniform frequency, the amplitude varies in response to the amount of frictional force produced by the viscidity between the sensor plates and the sample. To produce uniform amplitude, the vibro viscometer controls the electrical current that drives the vibration of the spring plates. Because the frictional force of viscidity is directly proportional to the viscosity, the driving electric current for vibrating the spring plates with a constant frequency to produce uniform amplitude is also directly proportional to the viscosity of each sample. The vibro viscometer measures the driving electric current, and then the viscosity is given by the positive correlation between the driving electric current and the viscosity.

Calibration

The viscometer can be calibrated using a standard viscosity fluid or a sample of a known viscosity. Calibration allows the viscometer to maintain the accuracy constantly. By calibrating an actual sample, using the viscosity value obtained by another type of viscometer as a correction value, the measurement data obtained by the SV series viscometer can be combined into those obtained by the other type of viscometer. Simplified calibration when measuring the viscosity near 1 mPa·s, (SV-10 only).Simplified calibration using purified water is a one-key operation. The SV-10 has a built-in function to measure the temperature of the purified water using the temperature sensor and calculates the viscosity value of the purified water at that temperature. At this time, be careful not to influence the viscosity value by generating bubbles. Standard windows communication tools WinCT-Viscosity Windows communication tools WinCT-Viscosity (CD-ROM) is provided as standard. The CD-ROM contains the graphing program RsVisco, which imports the data to a personal computer and displays the results as a graph in real time. With RsVisco, changes in viscosity over time and temperature dependency of viscosity can be observed easily and the obtained data can be saved in files.

Precautions

Install the viscometer in an environment where the temperature and humidity are not excessive. The best operating temperature is 25°C ± 2°C at 45-60% relative humidity.

For precise measurement, install the viscometer where there are no great changes in temperature and humidity.

Install the viscometer where it is not exposed to direct sunlight and it is not affected by heaters or air conditioners.

Install the viscometer where it is free of dust.

Install the viscometer away from equipment which produces magnetic fields.

The viscometer uses the Tuning-fork Vibration Method. So, use much care to avoid external vibration, especially when measuring low viscosity.
Protect the internal parts from liquid spills and excessive dust.
Do not disassemble the viscometer.
When precise measurement is required, acclimatize the viscometer to the measuring environment. After installation, plug in the AC adapter and warm up the viscometer for one hour or more.

During usage
To level the surface of the sample, adjust the leveling feet so that the center of the narrow part of the right and left sensor plates is on the liquid surface. The viscosity of a liquid is temperature dependent and changes by negative 2 to negative 10 percent, per degree Celsius. Take changes in the liquid temperature into consideration for an accurate measurement. Be sure to calibrate using the standard viscosity fluid or purified water before measurement. In a measurement that takes a long time, perform calibration periodically, as necessary. Placing the sensor plates and the temperature sensor in the sample may change the sample temperature. For precise measurement, leave the sample as is for a while, after placing the sensor plates and the temperature sensor, to ensure no changes to the sample temperature. And then, start a measurement. Ensure a stable power source when using the AC adapter. Use only your finger to press the keys. Using a sharp instrument such as a pen may damage keys. The sample cup is made of polycarbonate (PC) and is not appropriate for organic solvents. When organic solvents are used as a sample fluid, do not use the accessory sample cup. Use the glass sample cup (AX-SV-35) that is sold separately or a commercially-available glass beaker. The protector can be raised or removed. So, even when a beaker is used, the viscosity can be measured with a small amount of sample.

After use
Remove any residual sample material from the sensor plates, temperature sensor and protector using alcohol. Using the sensor plates, temperature sensor and protector with residue of an old sample left on will cause a measurement error. Clean the sensor plates carefully to avoid bending them. The sensor plates and the temperature sensor are made of stainless steel (SUS304). The surface is plated with 24K gold.

Note: Liquids with strong acidity may remove the gold plating and corrode the sensor plates and the temperature sensor.

Measurement
The SV Series Sine-wave Vibro Viscometer, as a measuring principle, detects the product of viscosity and density.
Displayed viscosity value = Viscosity × Density
While the displayed value has a unit of mPa·s, it indicates the product of viscosity and density
Example: When a sample has an absolute value of viscosity of 2.00 mPa·s and density of 1.000.
Displayed value = 2.00 [mPa·s] × 1.000 = 2.00 [mPa·s].

At measurement
Divide the displayed viscosity value by the sample density to obtain the absolute value of viscosity.
Example

1. Measure the sample and confirm the displayed viscosity value. Here, 736 mPa·s as an example.
2. Check the sample density at the temperature when the sample is measured. Here, 0.856 mg /cc as an example.
3. Divide the displayed viscosity value by the sample density to obtain the absolute value of viscosity.

Absolute viscosity value =
$\frac{\text{Displayed viscosity value}}{\text{Density value}} = \frac{736}{0.856} = 860$ mPa·s

Here, 860 mPa·s is obtained as the absolute viscosity value.

Rotational Viscometer
A motorized cylindrical rotor is inserted into a sample and rotated at a constant speed. The rotational viscometer employs the measurement method that assumes viscosity is directly proportional to a running torque required to produce a steady rotating motion. As shown in figure-2, when the rotation becomes steady, the running torque caused by the viscosity and the twist of the spring is balanced. The twist angle of the spring is proportional to the viscosity of the sample, and an index of this is displayed on the scale. Some devices display the digital value of the viscosity coefficient converted from running torque. The method shown in figure-2, the single cylindrical Brookfield rotational viscometer. The principle of operation of this Viscometer is the same as all other rotational viscometers: a spindle (cylinder or disk) is submerged in the sample to be tested,

measuring the force applied to overcome the resistance against rotation or flow. A spring is connected between the spindle (cylinder or disk) and the motor shaft which is rotating on a certain speed. The deviation angle of the spindle with respect to the measuring spring is measured electronically obtaining a torque value. The torque value measured with the Viscometer is based on the rotating speed and the geometry of the spindle; the result is a direct reading of the viscosity value in mPas/cP. Depending on the viscosity, the resistance to the movement of a substance changes proportionally to the speed or size of the spindle. The viscometer has been calibrated to obtain viscosity readings in mPas or cP considering speed and spindle type. The combination of different speeds and spindles allows optimal viscosity measurements within the wide range of the instrument.

Measurement

Inserting the spindle

If the selected spindle is of a disc type, it should be submerged carefully in sample to avoid bubbles forming under its bottom surface. To insert the spindles,(Figure-3) raise slightly the shaft holding it firmly with one hand and with the other hand screw the spindle. This operation must be done very carefully to make sure that the spindle is not bent and the shaft is not damaged. The spindle and its counterpart with the inner thread should be clean. Now the spindle can be immersed in the sample up to the immersion point, indicated with a groove on the same spindle. The shaft of the instrument should not be knocked against the sides of the container while the spindle is inserted since this might impair its vertical alignment.

Press **start** button to start a measurement. Stable flow conditions are reached quickly and the reading values of the Viscometer can be considered correct within few seconds (depending on the selected speed and the viscosity of the sample) .The message **"ERROR"** appearing on the screen indicates that the maximum viscosity value has been exceeded. In this case, the speed should be reduced or a larger spindle should be used. Pressing **stop** button, the instrument stops the motor, displaying the last measurement value. The rpm's will progressively decrease until zero rpm is reached, to protect the most delicate parts of the instrument. On pressing **start** button again, the viscometer will recover the preset speed value. To modify the spindle and rpm parameters, Press **Enter** button to return to the data screen. If viscosity value reading exceeds the optimum measuring range (<10% and >90% of selected full scale), the instrument will give a warning beep.

Capillary viscometer

In the below figure-4, Capillary viscometer, the laminar flow of liquid flows through a cylindrical capillary tube. You determine viscosity by measuring the flow rate of the fluid flowing through the capillary tube and the pressure differential between both ends of the capillary tube. This measurement method is based on the laws of physics; therefore, this is called the absolute measurement of viscosity. There is another type of capillary viscometer, made of glass; it has a simple principle and structure. Due to the simplicity of the principle, it has been used for a long time and has been greatly improved over the years. This capillary viscometer can obtain kinetic viscosity by measuring the time it takes for a certain amount of sample to flow by free-fall through the capillary tube. Each viscometer is given the viscosity constant, which was valued by calibrating with a Viscosity Standard Fluid. The principle and structure of the capillary viscometer is simple. However, to take accurate measurements, you must keep the inside of the capillary viscometer very clean. Before each measurement you must clean it using a cleaning liquid such as benzene, followed by another cleansing with acetone, then rinse using purified water. Also, a thorough drying of the capillary tube is required between each cleaning. Temperature control is also essential because glass is susceptible to thermal expansion or contraction under the influence of temperature, especially in lower viscosity ranges. These thermal impacts might introduce errors to the measurement. You must also measure the density of the measuring sample beforehand because the viscosity is given by calculating from the measured result acquired as kinetic viscosity.

Measurement

Capillary viscometer was taken and cleaned with water and acetone. Viscometer was fixed to a stand in a vertical position. Known quantity of water (15ml or 20ml) was transfer with the help of pipette through wide limb. Now water was sucked through the other limb up to a level higher than the upper mark ‘A’ and clamp the rubber tube with finger to stop the flow of water. Release the rubber tube and water was allowed to flow down. Stop clock was started when the water level first passes the upper mark ‘A’ when the water level pass down lower mark ‘B’ then the clock was stopped and noted down the time taken for the flow of sample between ‘A’ and ‘B’. Repeat

this process twice and calculate the time of flow. The Poiseuilles equation to calculate the viscosity is

$$V = \frac{\pi \Delta p r^4 t}{8 \eta L}$$

V: volume of the liquid
r: radius of vessel
t: time
η: coefficient of viscosity
Δp: change of pressure
L: vessel lenght

In the Poiseuilles equation, radius of the viscometer, length of the viscometer and pressure difference remains constant. Hence the equation becomes as follows for the determination of viscosity of unknown liquid

$$\frac{\eta 2}{\eta 1} = t2\rho 2/t1\ \rho 1\ [mPa{\cdot}s]$$

Where $\eta 1$= Viscosity of known liquid, t1 =time taken by known liquid, ρ1=density of known liquid. $\eta 2$= Viscosity of unknown liquid, t2= time taken by unknown liquid, ρ2= density of unknown liquid.

Falling-Ball Viscometer

The below figure-5 shows the falling-ball viscometer measures viscosity by dropping (free-fall) a column- or sphere-shaped rigid body with known dimensions and density into a sample and measuring the time taken for it to fall a specific distance.The principle for the viscosity measurement under the law of free-fall of a rigid body in the gravity field. Another type of device measures traveling time when horizontally transporting a rigid body, such as a piston, in a sample fluid at a constant speed by the force applied by the electromagnetic field. The principle of the viscometer is to determine the falling time of a ball of known diameter and density through a close to vertical glass tube of known diameter and length, filled with the fluid to be tested. The viscosity of the sample liquid is related to the time it takes for the ball to pass a distance between two specified lines on the cylindrical tube. Turning the measurement tube results in returning of the ball and it is possible to re-measure the time over the same distance. The result is dynamic viscosity with the standard dimension (mPa.s).Velocity of a ball which is falling through a liquid in a tube is dependent on the viscosity of the liquid.

When the ball moves through the liquid, it is affected by the gravity, buoyancy and frictional forces: Gravity as downward force, buoyancy and friction as the upward forces.

$$W = mg = V \rho s\ g = 4/3\pi r^3 \rho s g$$

ρs :density of ball, g: gravitational acceleration, V: volume of ball, r:raduis of ball.

Buoyant force, F1, acts upward and it is dependent of the density of the liquid which is displaced by the ball.

$$F1 = V \rho L\ g = 4/3\pi r^3 \rho L g$$

ρL =density of liquid.

The liquid has a dynamic viscosity, which produces a resistance against the ball movement. This frictional force is derived from the Stokes's law:

$F2 = 6\pi \eta r u$

u :velocity of the ball.

Whilst gravity and buoyant force are static and independent from the velocity, the frictional force raises with the velocity. Therefore, the velocity of the falling ball raises till the net forces is zero:

W-F1-F2=0

Combination of these equations would result in:

$$u = 2/9\ r^2\ g\ (\rho s - \rho L)/\eta$$

Equation shows that the viscosity of liquid (η) can be gained from the velocity of ball which is going down through this liquid. The studied liquid is in a glass tube which has two marks by distance L. In the experiment, the time it takes for liquid to pass through these two marks is measured. Modification of equation yields the constant coefficients are changed into a single coefficient K, which is viscometer constant and can be determined by using distilled water as it has well-known viscosity.

Measurement

Fill the falling tube with studied liquid and put in the ball cautiously. Add more liquid till no air bubbles can be seen. Then close the falling tube by its cap. Before starting the measurement, it is better to turn the falling tube up and down at least once in order to enhance temperature uniformity along the tube. Turn the falling tube 180 degree. Start stop watch when the ball reaches to the first marks on the tube, and measure the time between the two marks. For better and more accurate results it is recommended to repeat the measurement10 times at each temperature. After changing the bath temperature, it's highly recommended to wait at least 20 minutes to ensure temperature stability for the sample. At the end of the experiment, empty the tube from the liquid and remove the ball from the tube very carefully.

Clean the tube with suitable solvent and/or a brush. Write down the density of liquid and ball, ρL and ρs respectively. Calculate the average [t] for each temperature and calculate the viscosity using equation.

Note: The liquid in the falling tube should be free of bubbles. Generally the measurement for calibration is done at 20°C.Measuring of the time starts when the lower edge of the ball touches the upper mark and ends when crosses the lower mark.

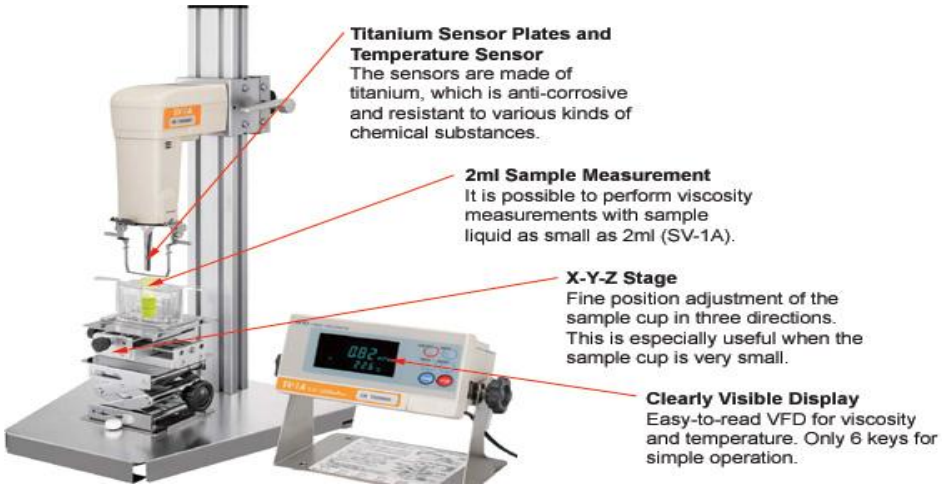

Fig. 1: Sine-wave Vibro Viscometer

Fig. 2: Brookfield rotational viscometer

Fig. 3: Spindles

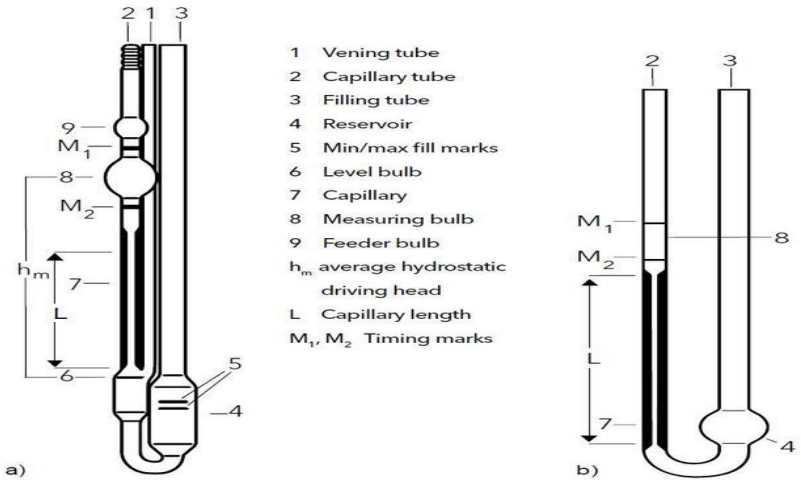

Fig. 4: Capillary viscometer

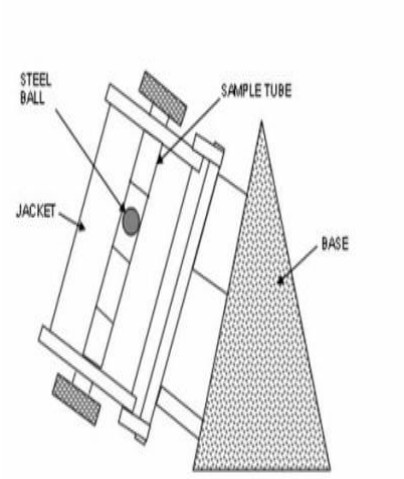

Fig. 5: Falling-Ball Viscometer

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